

## Trigonometrik ifadelerin Karşılaştırılması

\* Verilen trigonometrik ifadelerin I. Bölgeye karşılık gelen değeri bulunur.

\* Cosinüs varsa sinüse, cotangant varsa tanganta çevrilir.

\* 1. Bölgede  $\alpha > \beta$  olsun. O halde,  $\sin \alpha > \sin \beta$ ,  $\tan \alpha > \tan \beta$

\* 1. bölgedeki bir açı için her zaman  $\tan \alpha > \sin \alpha$

Çünkü,  $\tan \alpha = \frac{\text{Karşı}}{\text{Komşu}}$ ,  $\sin \alpha = \frac{\text{Karşı}}{\text{Hip.}}$

Hipotenüs > Komşu  $\rightarrow \tan \alpha > \sin \alpha$

$\sin x \rightarrow$

$\tan x \rightarrow$

Yani,  $a > b$ ,  $a = b$ ,  $a < 45^\circ$  ise,  $\tan a > \sin b$

**1985-Öys:**  $a = \sin 5$ ,  $b = \sin 85$ ,  $c = \sin 105$  ise  $a, b, c$  yi sıralayınız?

**Çözüm:**

- $a = \sin 5$
- $b = \sin 85$
- $c = \sin 105 = \sin 75$

$b > c > a$

$\sin 85 > \sin 75 > \sin 5$

**Örnek:**  $a = \sin 120$ ,  $b = \cos 50$ ,  $c = \tan 50$  ise  $a, b, c$  yi sıralayınız?

**Çözüm:**

- $\sin 120 = \sin 60$
  - $\cos 50 = \sin 40$
  - $\tan 50$
- $\tan 50 > \sin 60 > \sin 40$
- $c > a > b$

**Örnek:**  $a = \cos 330$ ,  $b = \sin 210$ ,  $c = \tan 70$   $d = \cot 250$  ise  $a, b, c, d$  yi sıralayınız?

**Çözüm:**

- $a = \cos 330 = \cos 30 = \sin 60$
- $b = \sin 210 = -\sin 30$
- $c = \tan 70$
- $d = \cot 250 = \cot 70 = \tan 20$

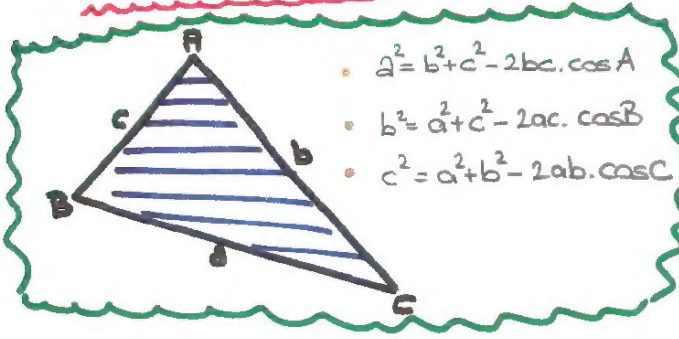
en büyük  $\rightarrow c$ , en küçük  $\rightarrow b$   
peki  $\sin 60$  mı  $\tan 20$  mi daha büyük?



✓ Sinüs için  $[0, 1]$  aralığı yaklaşık 90 parçaya bölünmüş ve  $60^\circ$  orta değerın sağında yani 1'e yakın olan taraftadır.

✗ tangant için  $[0, 1]$  aralığı yaklaşık 45 parçaya bölünmüş ve  $20^\circ$  orta değerın solunda yer almaktadır. Yani 0'a yakın olan taraftadır.  $\sin 60 > \tan 20$  o halde,  $c > a > d > b$

## # cosinüs Teoremi #



$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cdot \cos A \\ b^2 &= a^2 + c^2 - 2ac \cdot \cos B \\ c^2 &= a^2 + b^2 - 2ab \cdot \cos C \end{aligned}$$

**1971-Üss:**  $a^2 = b^2 + c^2 + bc$  ise  $\hat{A} = ?$

**Çözüm:**

$a^2 = b^2 + c^2 + bc \rightarrow$  Soruda verilen

$- / a^2 = b^2 + c^2 - 2bc \cdot \cos A \rightarrow$  cosinüs teoremi

$0 = 2bc \cdot \cos A + bc, -bc = 2bc \cdot \cos A$

$-\frac{1}{2} = \cos A \rightarrow \hat{A} = 120^\circ$

**Örnek:** Bir üçgenin kenarları  $a, b, c$  ve  $a^2 = b^2 + c^2 - \sqrt{2}bc$  ise  $m(\hat{A})$  kaçtır?

**Çözüm:**

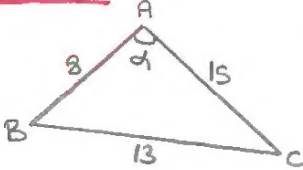
$a^2 = b^2 + c^2 - \sqrt{2}bc \rightarrow$  Soruda verilen

$- / a^2 = b^2 + c^2 - 2bc \cdot \cos A \rightarrow$  cosinüs teoremi

$0 = 2bc \cdot \cos A - \sqrt{2}bc, \sqrt{2}bc = 2bc \cdot \cos A$

$\frac{\sqrt{2}}{2} = \cos A \rightarrow \hat{A} = 45^\circ$

**Örnek:**



$\alpha$  kaç derecedir?

**Çözüm:**

$13^2 = 8^2 + 15^2 - 2 \cdot 8 \cdot 15 \cdot \cos \alpha$

$169 = 64 + 225 - 240 \cdot \cos \alpha$

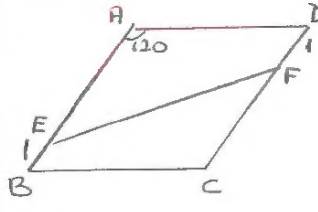
$240 \cdot \cos \alpha = 289 - 169$

$240 \cos \alpha = 120, \cos \alpha = \frac{1}{2}$

$\alpha = 60^\circ$

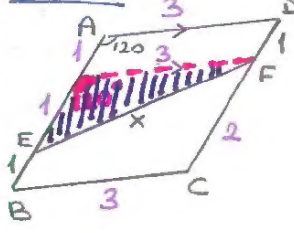
Sayfa: 12

**1980-Üss:**



ABCD eşkenar dörtgen  
 $|AB|=3$  ise  $|EF|=?$

**Çözüm:**



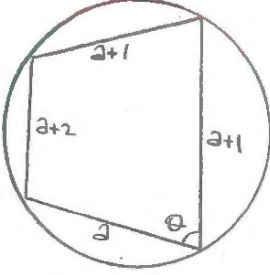
$x^2 = 1^2 + 3^2 - 2 \cdot 1 \cdot 3 \cdot \cos 120$

$x^2 = 1 + 9 - 6 \cdot (-\frac{1}{2})$

$x^2 = 10 + 3$

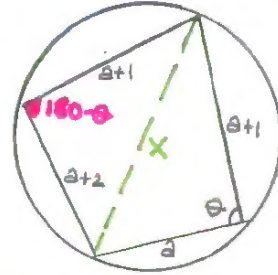
$x^2 = 13, x = \sqrt{13}$

**1975-Üss:**



$\cos \theta$  kaçtır?

**Çözüm:**



Kirişler dörtgeninden  
 $\theta$  nin karşısı  $180 - \theta$  dir.

\* İki tane cosinüs  
teoremi uygulanıp  
taraf tarafa toplanır.

$x^2 = a^2 + (a+1)^2 - 2 \cdot a \cdot (a+1) \cdot \cos \theta$

$x^2 = (a+1)^2 + (a+2)^2 - 2(a+1) \cdot (a+2) \cdot \cos(180 - \theta)$

$\cos(180 - \theta) = -\cos \theta$

$0 = -a^2 + (a+2)^2 + 2 \cdot a \cdot (a+1) \cdot \cos \theta + 2 \cdot (a+1) \cdot (a+2) \cdot (-\cos \theta)$

$0 = -a^2 + 4a + 4 + 2 \cdot (a+1) \cdot \cos \theta \cdot (a+a+2)$

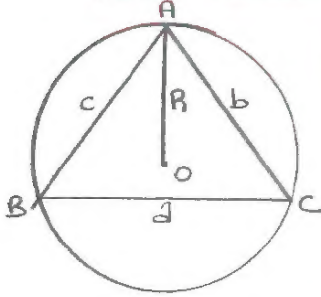
$0 = 4(a+1) + 2 \cdot (a+1) \cdot \frac{(2a+2)}{2(a+1)} \cdot \cos \theta$

$-4(a+1) = 4(a+1) \cdot \cos \theta$

$-1 = (a+1) \cdot \cos \theta, \cos \theta = \frac{-1}{a+1}$

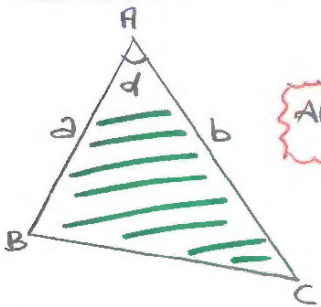


## # Sinüs Teoremi #



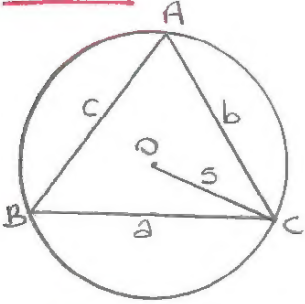
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

## Sinüs ile Alan Teoremi:



$$A(\triangle ABC) = \frac{1}{2} \cdot a \cdot b \cdot \sin C$$

## Örnek:



$$\sin \hat{A} + \sin \hat{B} + \sin \hat{C} = \frac{3}{2}$$

İse  $A(\triangle ABC)$  kaçtır?

Çözüm:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = 2 \cdot 5 = 10$

$$a = 10 \cdot \sin \hat{A} \rightarrow \sin \hat{A} = \frac{a}{10}$$

$$b = 10 \cdot \sin \hat{B} \rightarrow \sin \hat{B} = \frac{b}{10}$$

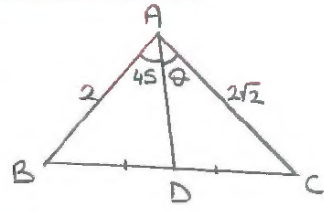
$$c = 10 \cdot \sin \hat{C} \rightarrow \sin \hat{C} = \frac{c}{10}$$

$$\sin \hat{A} + \sin \hat{B} + \sin \hat{C} = \frac{3}{2} \text{ ise,}$$

$$\frac{a}{10} + \frac{b}{10} + \frac{c}{10} = \frac{3}{2}, \quad \frac{a+b+c}{10} = \frac{3}{2}$$

$$a+b+c = \frac{30}{2} = 15, \quad A(\triangle ABC) = 15$$

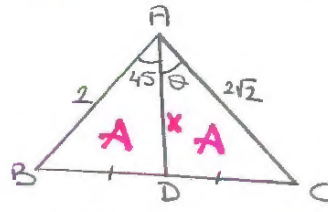
1991-ÖYS:



$$\sin \theta = ?$$

## Çözüm:

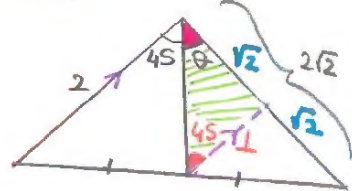
1. Yol:  $|BD| = |DC|$  ise,  $A(\triangle ABD) = A(\triangle ADC)$



$$\frac{1}{2} \cdot 2 \cdot X \cdot \sin 45 = \frac{1}{2} \cdot X \cdot 2\sqrt{2} \cdot \sin \theta$$

$$\frac{\sqrt{2}}{2} = \sqrt{2} \cdot \sin \theta, \quad \sin \theta = \frac{1}{2}$$

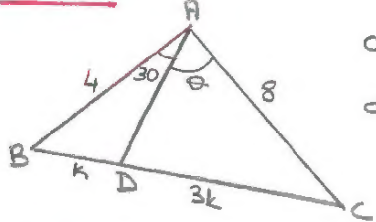
2. Yol:



Taralı üçgende,

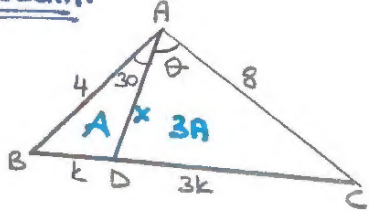
$$\frac{1}{\sin \theta} = \frac{\sqrt{2}}{\sin 45}, \quad \sqrt{2} \cdot \sin \theta = \frac{\sqrt{2}}{2}, \quad \sin \theta = \frac{1}{2}$$

## Örnek:



$\cos \theta$  nin pozitif değeri kaçtır?

## Çözüm:

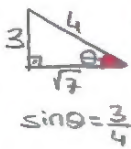


$$\frac{1}{2} \cdot 4 \cdot X \cdot \sin 30 = A$$

$$3 \sin 30 = 2 \sin \theta$$

$$\frac{1}{2} \cdot 4 \cdot 8 \cdot \sin \theta = 3A$$

$$3 \cdot \frac{1}{2} = 2 \sin \theta, \quad \sin \theta = \frac{3}{4}$$



$$\cos \theta = \frac{\sqrt{7}}{4}$$

$$\sin \theta = \frac{3}{4}$$

## Bilinmesi Gereken Trigonometri Formülleri :

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin 2a = 2 \sin a \cos a$$

$$\cos 2a = \cos^2 a - \sin^2 a$$

$$\cos^2 a + \sin^2 a = 1$$

$$\cos^2 a = 1 - \sin^2 a \quad \sin^2 a = 1 - \cos^2 a$$

$$\cos a \cos b = \frac{\cos(a+b) + \cos(a-b)}{2}$$

not:  $\cos a \cos b$  bilirse, " $\sin a \cos b$ ", " $\sin b \cos a$ " ve " $\sin a \sin b$ " formüllerini ezberlemek gereksizdir.

$$\sin a + \sin b = 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$\sin a - \sin b = 2 \sin\left(\frac{a-b}{2}\right) \cos\left(\frac{a+b}{2}\right)$$

not:  $\sin a + \sin b$  ve  $\sin a - \sin b$  bilirse " $\cos a + \cos b$ ", " $\cos a - \cos b$ " formüllerini ezberlemek gereksizdir.

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}, \quad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$$

$$\cot(a+b) \rightarrow \frac{1}{\tan(a+b)}$$

$$\cot(a-b) \rightarrow \frac{1}{\tan(a-b)}$$

$$\cot 2a = \frac{1}{\tan 2a}$$

$$\# \sin(a+b) = \sin a \cos b + \sin b \cos a \#$$

örnek:  $\sin 75$  in değeri nedir?

$$\begin{aligned} \text{Çözüm: } \sin(45+30) &= \sin 45 \cos 30 + \sin 30 \cos 45 \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\text{örnek: } \frac{\sin 10 \cos 30 + \sin 30 \cos 10}{\sin 90 \cos 50 + \cos 90 \sin 50} = ?$$

$$\text{Çözüm: } \frac{\sin(10+30)}{\sin(90+50)} = \frac{\sin 40}{\sin 140} = \frac{\sin 40}{\sin 40}$$

$$\Rightarrow 1$$

$$\# \sin(a-b) = \sin a \cos b - \sin b \cos a \#$$

$$\text{örnek: } \sin 72 \cos 27 - \sin 27 \cos 72 = ?$$

$$\text{Çözüm: } \sin(72-27) = \sin 45 = \frac{\sqrt{2}}{2}$$

$$\# \cos(a+b) = \cos a \cos b - \sin a \sin b \#$$

$$\text{örnek: } \cos 105 \text{ in değeri nedir?}$$

$$\begin{aligned} \text{Çözüm: } \cos(60+45) &= \cos 60 \cos 45 - \sin 60 \sin 45 \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

$$\text{örnek: } \cos(3x+y) \text{ ifadesini açınız?}$$

$$\text{Çözüm: } \cos 3x \cos y - \sin 3x \sin y$$

$$\# \cos(a-b) = \cos a \cos b + \sin a \sin b \#$$

$$\text{örnek: } \cos 20 \cos 50 + \sin 20 \sin 50 = ?$$

$$\begin{aligned} \text{Çözüm: } \cos(20-50) &= \cos(-30) \\ &= \cos 30 = \sin 60 = \frac{\sqrt{3}}{2} \end{aligned}$$



#  $\sin 2\alpha = 2 \sin \alpha \cos \alpha$  #

**örnek:**  $\sin x \cdot \cos x$  ifadesinin en sade halini bulunuz?

**Çözüm:**  $\sin x \cdot \cos x = \frac{2 \sin x \cos x}{2} = \frac{\sin 2x}{2}$

**örnek:**  $\sin 15 \cos 15$  değeri kaçtır?

**Çözüm:**  $\sin 15 \cos 15 = \frac{2 \sin 15 \cos 15}{2} = \frac{\sin 30}{2} = \frac{1}{4}$

**örnek:**  $\cos 20 \cdot \cos 40 \cdot \cos 80 = ?$

**Çözüm:**

$$\frac{2 \sin 40 \cdot \cos 20 \cdot \cos 40 \cdot \cos 80}{2 \sin 20} = \frac{2 \sin 80 \cdot \cos 80}{2 \cdot 4 \sin 20} = \frac{\sin 160}{8 \sin 20} = \frac{\sin 20}{8 \sin 20} = \frac{1}{8}$$

**örnek:**  $\cos 20 \cdot \cos 40 \cdot \cos 60 \cdot \cos 80 = ?$

**Çözüm:**  $\frac{\cos 20 \cos 40 \cos 80 \cdot \cos 60}{\frac{1}{8} \cdot \frac{1}{2}} = \frac{1}{16}$

**örnek:**  $\cos \frac{\pi}{7} \cdot \cos \frac{2\pi}{7} \cdot \cos \frac{4\pi}{7} = ?$

**Çözüm:**

$$\frac{2 \sin \frac{4\pi}{7} \cdot \cos \frac{\pi}{7} \cdot \cos \frac{2\pi}{7} \cdot \cos \frac{4\pi}{7}}{2 \sin \frac{4\pi}{7}} = \frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}} = \frac{\sin(\pi + \frac{\pi}{7})}{8 \sin \frac{\pi}{7}} = \frac{-\sin \frac{\pi}{7}}{8 \sin \frac{\pi}{7}} = -\frac{1}{8}$$

Sayfa: C15

**2011-LYS:**  $0 < x < \frac{\pi}{2}$  olmak üzere,

$\cot x - 3 \tan x = \frac{1}{\sin 2x}$  ise  $\sin^2 x$  kaçtır?

**Çözüm:**  $\frac{\cos x}{\sin x} - \frac{3 \sin x}{\cos x} = \frac{1}{2 \sin x \cos x}$

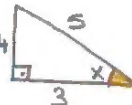
$\frac{\cos^2 x - 3 \sin^2 x}{\sin x \cos x} = \frac{1}{2 \sin x \cos x}$

$(1 - \sin^2 x) - 3 \sin^2 x = \frac{1}{2}, 1 - 4 \sin^2 x = \frac{1}{2}$

$1 - \frac{1}{2} = 4 \sin^2 x, 4 \sin^2 x = \frac{1}{2}, \sin^2 x = \frac{1}{8}$

**1983-ÖYS:**  $0 < x < \frac{\pi}{2}$ ,  $\tan x = \frac{4}{3}$  ise

$\frac{\sin^3 x - \cos^3 x}{1 + \frac{1}{2} \sin 2x}$  ifadesinin değeri kaçtır?

**Çözüm:**   $\tan x = \frac{4}{3}$

$\frac{(\frac{4}{5})^3 - (\frac{3}{5})^3}{1 + \frac{1}{2} \cdot \frac{4}{5} \cdot \frac{3}{5}} = \frac{\frac{64-27}{125}}{1 + \frac{12}{25}} = \frac{\frac{37}{125}}{\frac{37}{25}} = \frac{1}{5}$

**2010-LYS:**  $\frac{\tan 60}{\sin 20} - \frac{1}{\cos 20} = ?$

**Çözüm:**  $\frac{\frac{\sin 60}{\cos 60}}{\sin 20} - \frac{1}{\cos 20} = \frac{\sin 60}{\sin 20 \cos 60} - \frac{1}{\cos 20}$

$$= \frac{\sin 60 \cos 20 - \sin 20 \cos 60}{\sin 20 \cos 20 \cos 60} = \frac{\sin(60-20)}{\sin 20 \cos 20 \cos 60} = \frac{\sin 40}{\sin 20 \cos 20 \cos 60} = \frac{2 \sin 20 \cos 20}{\sin 20 \cos 20 \cdot \frac{1}{2}} = 4$$


**2008-ÖSS:**  $\sin 2x = a$  ise  $(\sin x + \cos x)^2 = ?$

**Çözüm:**  $\underbrace{\sin^2 x + \cos^2 x}_1 + \underbrace{2\sin x \cos x}_{\sin 2x} = 1 + a$

**2007-ÖSS:**  $(\sin \frac{\pi}{12} + \cos \frac{\pi}{12})^2 = ?$

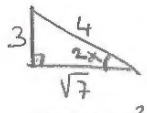
**Çözüm:**  $(\sin 15 + \cos 15)^2 = \underbrace{\sin^2 15 + \cos^2 15}_1 + \underbrace{2\sin 15 \cos 15}_{\sin 30}$   
 $= 1 + \frac{1}{2} = \frac{3}{2}$

**1972-ÜSS:**  $\tan x = 2$  ise  $\sin 2x$  nedir?

**Çözüm:**   $\sin 2x = 2 \sin x \cos x$   
 $\tan x = 2$   $\sin 2x = 2 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{4}{5}$

**1994-ÖYS:**  $\cos x - \sin x = \frac{1}{2}$  ise  $\cos 2x = ?$

**Çözüm:**  $(\cos x - \sin x)^2 = (\frac{1}{2})^2$   
 $\Rightarrow \underbrace{\cos^2 x + \sin^2 x}_1 - \underbrace{2\sin x \cos x}_{\sin 2x} = \frac{1}{4}$   
 $\Rightarrow 1 - \frac{1}{4} = \sin 2x, \sin 2x = \frac{3}{4}$

  $\cos 2x = \frac{\sqrt{7}}{4}$   
 $\sin 2x = \frac{3}{4}$


**1967-ÜSS:**  $\sin x - \cos x = \frac{1}{2}$  ise  $\sin 2x = ?$

**Çözüm:**  $(\sin x - \cos x)^2 = (\frac{1}{2})^2$   
 $\underbrace{\sin^2 x + \cos^2 x}_1 - \underbrace{2\sin x \cos x}_{\sin 2x} = \frac{1}{4}$

$1 - \frac{1}{4} = \sin 2x, \sin 2x = \frac{3}{4}$

#  $\cos 2a = \cos^2 a - \sin^2 a$  #

**2011-LYS:**  $\cos x = \frac{-4}{5}$  ise  $\cos 2x = ?$

**Çözüm:**   $\cos x = \frac{-4}{5}$

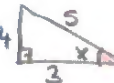
$\cos 2x = \cos^2 x - \sin^2 x = (\frac{4}{5})^2 - (\frac{3}{5})^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

**1989-ÖYS:**  $\cos 36 = \frac{\sqrt{5}+1}{4}$  ise  $\cos 72 = ?$

**Çözüm:**  $\cos 72 = \cos(36+36) = \cos^2 36 - \sin^2 36$   
 $= \cos^2 36 - (1 - \cos^2 36) = 2\cos^2 36 - 1$   
 $= 2 \cdot (\frac{\sqrt{5}+1}{4})^2 - 1 = 2 \cdot (\frac{5+1+2\sqrt{5}}{16}) - 1$   
 $= \frac{6+2\sqrt{5}-8}{8} = \frac{2\sqrt{5}-2}{8} = \frac{\sqrt{5}-1}{4}$

**2010-LYS:**  $3\sin x - 4\cos x = 0, |\cos 2x| = ?$

**Çözüm:**  $3\sin x = 4\cos x, \frac{\sin x}{\cos x} = \frac{4}{3}$

$\tan x = \frac{4}{3}$    
 $|\cos 2x| = |\cos^2 x - \sin^2 x| = |(\frac{3}{5})^2 - (\frac{4}{5})^2|$   
 $= |\frac{9}{25} - \frac{16}{25}| = |\frac{-7}{25}| = \frac{7}{25}$

**2006-ÖSS:**  $\frac{\sin 2a}{1 - \cos 2a}$  sonucu nedir?

**Çözüm:**  $\frac{2\sin a \cos a}{1 - (\cos^2 a - \sin^2 a)} = \frac{2\sin a \cos a}{1 - \cos^2 a + \sin^2 a}$   
 $= \frac{2\sin a \cos a}{2\sin^2 a} = \frac{2\sin a \cos a}{2\sin a \sin a} = \cot a$

**2007-Üss:**  $\frac{\cos 2a}{1 - \tan^2 a} = ?$

**Çözüm:**  $\frac{\cos^2 a - \sin^2 a}{1 - \frac{\sin^2 a}{\cos^2 a}} = \frac{\cos^2 a - \sin^2 a}{\frac{\cos^2 a - \sin^2 a}{\cos^2 a}}$   
 $= \frac{\cos^2 a - \sin^2 a}{1} \cdot \frac{\cos^2 a}{\cos^2 a - \sin^2 a} = \boxed{\cos^2 a}$

**1981-Öys:**  $\tan x = \frac{\sin 2y}{1 - \cos 2y}$  ise  $x + y = ?$

**Çözüm:**  $\frac{\sin 2y}{1 - \cos 2y} = \frac{2 \sin y \cos y}{2 \sin y \sin y} = \cot y$

$\tan x = \cot y$  ise  $\boxed{x + y = 90^\circ}$

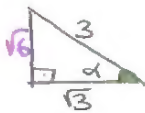
**1973-Üss:**  $\tan x = \frac{\sqrt{3}}{2}$  ise  $\cos 2x = ?$

**Çözüm:**   $\tan x = \frac{\sqrt{3}}{2}$

$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{2}{\sqrt{7}}\right)^2 - \left(\frac{\sqrt{3}}{\sqrt{7}}\right)^2 = \boxed{\frac{1}{7}}$

**1974-Üss:**  $\cos \alpha = \frac{\sqrt{3}}{3}$  ise  $0 < \alpha < \frac{\pi}{2}$

İçin  $\cos 2\alpha$  kaçtır?

**Çözüm:** 

$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{\sqrt{3}}{3}\right)^2 - \left(\frac{\sqrt{6}}{3}\right)^2 = \boxed{-\frac{1}{3}}$

**Örnek:**  $\cos 33 = a$  ise  $\cos 66 = ?$

**Çözüm:**  $\cos 66 = \cos^2 33 - \sin^2 33$   
 $\cos 66 = 2\cos^2 33 - 1 = \boxed{2a^2 - 1}$

**Örnek:**  $\sin 55 = m$  ise  $\sin 20$  kaçtır?

**Çözüm:**  $\sin 55 = \cos 35 = m$   
 $\sin 20 = \cos 70 = ?$

$\cos 70 = \cos^2 35 - \sin^2 35 = \cos^2 35 - (1 - \cos^2 35)$   
 $= 2\cos^2 35 - 1 = \boxed{2m^2 - 1}$

**Örnek:**  $\sqrt{\frac{1 - \cos 66}{2}} = ?$

**Çözüm:**  $\sqrt{\frac{1 - (\cos^2 33 - \sin^2 33)}{2}}$   
 $= \sqrt{\frac{1 - \cos^2 33 + \sin^2 33}{2}} = \sqrt{\frac{\sin^2 33 + \sin^2 33}{2}}$   
 $= \sqrt{\frac{2\sin^2 33}{2}} = \sqrt{\sin^2 33} = \boxed{\sin 33}$

**Örnek:**  $\cos 64 = a$  ise  $\sin 8 = ?$

**Çözüm:**  $\sin 8 = \cos 32 = ?$

$\cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1$

$\cos 64 = 2\cos^2 32 - 1$  ,  $a + 1 = 2\cos^2 32$

$\cos^2 32 = \frac{a + 1}{2}$  ,  $\cos 32 = \sqrt{\frac{a + 1}{2}}$

$\sin 8 = \boxed{\sqrt{\frac{a + 1}{2}}}$



$$\# \cos a \cdot \cos b = \frac{\cos(a+b) + \cos(a-b)}{2} \#$$

**Örnek:**  $\cos 15 \cdot \cos 75 = ?$

**Çözüm:**  $\frac{\cos(15+75) + \cos(15-75)}{2}$

$$\Rightarrow \frac{\cos 90 + \cos(-60)}{2} = \frac{\cos(-60)}{2} = \frac{\cos 60}{2} = \frac{1}{4}$$

**Örnek:**  $\cos 75 \cdot \sin 15 = ?$

**Çözüm:**  $\cos 75 \cdot \sin 45 = \cos 75 \cdot \cos 45$

$$\Rightarrow \frac{\cos(75+45) + \cos(75-45)}{2} = \frac{\cos 120 + \cos 30}{2}$$

$$\Rightarrow \frac{-\cos 60 + \cos 30}{2} = \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}-1}{4}$$

**Örnek:**  $\sin 105 \cdot \sin 165 = ?$

**Çözüm:**  $\sin 75 \cdot \sin 15 = \cos 15 \cdot \cos 75$

$$\Rightarrow \frac{\cos(15+75) + \cos(15-75)}{2} = \frac{\cos 90 + \cos(-60)}{2}$$

$$= \frac{\cos 60}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

**Örnek:**  $\cos 3x \cdot \cos x = ?$

**Çözüm:**  $\frac{\cos(3x+x) + \cos(3x-x)}{2} = \frac{\cos 4x + \cos 2x}{2}$

**Örnek:**  $\cos 4x \cdot \sin 2x = ?$

**Çözüm:**  $\cos 4x \cdot \cos(90-2x)$

$$\Rightarrow \frac{\cos(4x+90-2x) + \cos(4x-(90-2x))}{2}$$

$$\Rightarrow \frac{\cos(90+2x) + \cos(6x-90)}{2}$$

$$\frac{\cos(90+2x) + \cos(270+6x)}{2} = \frac{-\sin 2x + \sin 6x}{2}$$

**Örnek:**  $\sin 3x \cdot \sin 5x = ?$

**Çözüm:**  $\cos(90-3x) \cdot \cos(90-5x)$

$$\Rightarrow \frac{\cos(90-3x+90-5x) + \cos(90-3x-(90-5x))}{2}$$

$$\Rightarrow \frac{\cos(180-8x) + \cos(2x)}{2} = \frac{-\cos 8x + \cos 2x}{2}$$

**Sonuç:** Sadece  $\cos a \cos b$  formülü ile "sina.cosb", "sinb.cosa", "sina.sinb" formüllerine gerek kalmadan sonuç bulunabilir.

$$\# \sin a + \sin b = 2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right) \#$$

**Örnek:**  $\sin 15 + \cos 15 = ?$

**Çözüm:**  $\sin 15 + \sin 75 = 2 \cdot \sin\left(\frac{15+75}{2}\right) \cdot \cos\left(\frac{15-75}{2}\right)$

$$\Rightarrow 2 \cdot \sin 45 \cdot \cos(-30) = 2 \cdot \sin 45 \cdot \cos 30$$

$$\Rightarrow 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$$

**Örnek:**  $\cos 3x + \cos x = ?$

**Çözüm:**  $\sin(90-3x) + \sin(90-x)$

$$\Rightarrow 2 \cdot \sin\left(\frac{90-3x+90-x}{2}\right) \cdot \cos\left(\frac{90-3x-(90-x)}{2}\right)$$

$$\Rightarrow 2 \cdot \sin(90-2x) \cdot \cos(-x)$$

$$\Rightarrow 2 \cdot \cos 2x \cdot \cos x$$

$$\# \sin a - \sin b = 2 \cdot \sin\left(\frac{a-b}{2}\right) \cdot \cos\left(\frac{a+b}{2}\right) \#$$

**Örnek:**  $\sin 105 - \sin 15 = ?$

**Çözüm:**  $2 \cdot \sin\left(\frac{105-15}{2}\right) \cdot \cos\left(\frac{105+15}{2}\right)$

$$\Rightarrow 2 \cdot \sin 45 \cdot \cos 60 = 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}$$



**2010-LYS:**  $\frac{1 + \cos 40}{\cos 55 \cdot \cos 35} = ?$

**Çözüm:**  $1 = \cos 0$  yazılır.

$$\frac{\cos 0 + \cos 40}{\cos 55 \cdot \cos 35} = \frac{\sin 90 + \sin 50}{\cos 55 \cdot \cos 35}$$

$$= \frac{2 \cdot \sin\left(\frac{90+50}{2}\right) \cdot \cos\left(\frac{90-50}{2}\right)}{\cos(55+35) + \cos(55-35)}$$

$$= \frac{2 \cdot \sin 70 \cdot \cos 20}{\cos 90 + \cos 20} = \frac{2 \cdot \sin 70 \cdot \cos 20}{1 + \cos 20}$$

$$= 4 \sin 70 \Rightarrow \boxed{4 \cdot \cos 20^\circ}$$

**1985-ÖYS:**  $\frac{1}{\sin 15} + \frac{1}{\cos 15} = ?$

**Çözüm:**  $\frac{1}{\sin 15} + \frac{1}{\cos 15} = \frac{\cos 15 + \sin 15}{\sin 15 \cdot \cos 15}$

$$= \frac{\sin 75 + \sin 15}{\frac{2 \cdot \sin 15 \cos 15}{2}} = \frac{2 \cdot \sin\left(\frac{75+15}{2}\right) \cdot \cos\left(\frac{75-15}{2}\right)}{\frac{\sin 30}{2}}$$

$$= \frac{2 \cdot \sin 45 \cdot \cos 30}{\frac{\sin 30}{2}} = \frac{2 \cdot \sin 45 \cdot \cos 30}{1} \cdot \frac{2}{\sin 30}$$

$$= \frac{2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \cdot 2}{\frac{1}{2}} = \boxed{2\sqrt{6}}$$

**1991-ÖYS:**  $\frac{\sin 3x}{\sin x} + \frac{\cos 3x}{\cos x} = 1$  ise  $\cos^2 x = ?$

**Çözüm:**  $\frac{\sin 3x}{\sin x} + \frac{\cos 3x}{\cos x} = \frac{\sin 3x \cdot \cos x + \cos 3x \cdot \sin x}{\sin x \cdot \cos x}$

$$\Rightarrow \frac{\sin(3x+x)}{\frac{2 \cdot \sin x \cos x}{2}} = \frac{\sin 4x}{\frac{\sin 2x}{2}} = \frac{2 \cdot \sin 4x}{\sin 2x} = \frac{2 \cdot 2 \cdot \sin 2x \cos 2x}{\sin 2x}$$

$$\Rightarrow 4 \cdot \cos 2x = 1 \text{ ise } \cos 2x = \frac{1}{4}$$

$$\cos^2 x - \sin^2 x = \frac{1}{4}, \quad 2\cos^2 x - 1 = \frac{1}{4}$$

$$\cos^2 x = \frac{5}{8}$$

**2007-ÖSS:**  $\frac{\sin 10 \cdot \cos 40 + \cos 10 \cdot \sin 40}{\cos 50 \cdot \cos 10 + \sin 50 \cdot \sin 10} = ?$

**Çözüm:**  $\frac{\sin(10+40)}{\cos(50-10)} = \frac{\sin 50}{\cos 40} = \boxed{1}$

**1996-ÖYS:**  $\frac{\sin 2A + \sin 4A}{\cos 2A + \cos 4A} = ?$

**Çözüm:**  $\frac{\sin 2A + \sin 4A}{\sin(90-2A) + \sin(90-4A)}$

$$= \frac{2 \cdot \sin\left(\frac{2A+4A}{2}\right) \cdot \cos\left(\frac{2A-4A}{2}\right)}{\sin(90-2A) + \sin(90-4A)}$$

$$= \frac{2 \cdot \sin\left(\frac{90-2A+90-4A}{2}\right) \cdot \cos\left(\frac{90-2A-(90-4A)}{2}\right)}{\sin(90-2A) + \sin(90-4A)}$$

$$= \frac{\sin 3A \cdot \cos(-A)}{\sin(90-3A) \cdot \cos A} = \frac{\sin 3A}{\cos 3A} = \boxed{\tan 3A}$$

**1996-ÖYS:**  $0 < \alpha < 90^\circ$

$$\frac{\sqrt{3} \cdot \sin 5 \cdot \cos 7 + \sqrt{3} \cdot \cos 5 \cdot \sin 7}{4 \cdot \cos 84 \cdot \cos 6} = \sin \alpha, \quad \alpha = ?$$

**Çözüm:**  $\frac{\sqrt{3} (\sin 5 \cdot \cos 7 + \cos 5 \cdot \sin 7)}{2 \cdot 2 \cdot \sin 6 \cdot \cos 6} = \sin \alpha$

$$= \frac{\sqrt{3} \cdot \sin 12}{2 \cdot \sin 12} = \sin \alpha, \quad \alpha = \boxed{60^\circ}$$

**1972-ÜSS:**  $\frac{\sin x + \sin 3x}{\cos x - \cos 3x} = ?$

**Çözüm:**  $\frac{\sin x + \sin 3x}{\sin(90-x) - \sin(90-3x)}$

$$\Rightarrow \frac{2 \cdot \sin\left(\frac{x+3x}{2}\right) \cdot \cos\left(\frac{x-3x}{2}\right)}{\sin(90-x) - \sin(90-3x)}$$

$$= \frac{2 \cdot \sin\left(\frac{90-x-(90-3x)}{2}\right) \cdot \cos\left(\frac{90-x+90-3x}{2}\right)}{\sin(90-x) - \sin(90-3x)}$$

$$\Rightarrow \frac{\sin 2x \cdot \cos(-x)}{\sin x \cdot \cos(90-2x)}$$

$$\Rightarrow \boxed{\cot x}$$